

# XII International Seminar on Optimization and Related Areas

October 5 - 9, 2015

## Welcome

Dear participants,

First of all, we want to welcome you all to the Twelfth International Seminar on Optimization and Related Areas - ISORA, as always, in memory of Eugen Blum.

- We would also like to thank to our sponsors, which made possible to organize this event.
- Universidad Nacional de Ingeniería. Facultad de Ciencias e Instituto General de Investigación.
- Ambassade de France au Pérou.
- ProUNI.
- Instituto Nacional de Matemática Pura e Aplicada.
- Fondo Nacional de Ciencia y Tecnología FONDECYT

Finally, we would like to thank you for participating in this version of ISORA, and for visiting us. We hope you enjoy your stay in Lima!

XII ISORA Organizing Committee Lima, October 5th, 2015

# **General Information**

### **Conference Venue**

The XII ISORA will take place at IMCA's auditorium. IMCA (Instituto de Matemática y Ciencias Afines, Alberto Benavides de la Quintana) is located in a quiet residential place: Calle Los Biólogos 245, Urbanización San César, in La Molina district.



- IMCA's auditorium has WiFi Internet, a personal computer, a multimedia projector and a large blackboard.
- ISORA participants must register with Ms Iris German, IMCA director's secretary. Ms German's office is located in the second floor of the building
- Coffee breaks will be served in the hall, next to the entrance of the auditorium.
- There are many restaurants near IMCA, most of them located on Constructores avenue (from IMCA's main entrance, to the right).
- Event dinner: This dinner will take place Thursday 8, at 20:00, at a restaurant we will announce soon.

### **Scientific Committee**

- Alfredo Iusem (IMPA, Brasil)
- Aris Daniilidis (CMM, Chile)
- Lionel Thibault (University of Montpellier 2, France)
- Marco Antonio López Cerdá (Universidad de Alicante, España)
- Frédéric Bonnans (INRIA Saclay, France)

### **Organizing Committee**

- Eladio Ocaña (chair), UNI, IMCA
- John Cotrina, UP & GO-IMCA
- Orestes Bueno, UP & GO-IMCA
- Oswaldo Velásquez, UNI, IMCA
- Roger Metzger, UNI, IMCA
- Yboon García, UP, UNI, GO-IMCA

# Schedule

	Monday 5	Tuesday 6	Wednesday 7	Thursday 8	Friday 9		
09:00 - 09:30		С9	C18	C26			
		A. Hantoute	W. Sosa	R. Luke			
09:30 - 10:00	C1	C10 C19		C27			
	M. Théra	T. Zakaryan	J. Peypouquet	M. de Lara			
10:00 - 10:20	COFEE BREAK						
10:20 - 10:50	C2	C11	C20	C28			
	J.B. Baillon	J. López	M. Quincampoix	V. Sessa			
10:50 - 11:40	P1	P3	P5	P7			
	A. Daniilidis	B. Fux Svaiter	H. Frankowska	S.D. Flåm	$\circ$		
12:00 - 14:00	L U N C H						
14:00 - 14:30	C3	C12	C21	C29			
	G. López-Acedo	U. Kohlenbach	R. Behling	W. de Oliveira			
14:30 - 15:00	C4	C13	C22	C30			
	R. Correa	A. Quilliot	R. López	C. Tammer	ア		
15:00 - 15:20	COFEE BREAK						
15:20 - 15:50	C5	C14	C23	<b>P8</b> (15:20 - 16:10)			
	E.R. Csenek	M. Lassonde	J.J. Rückmann	S. Simons			
15:50 - 16:20	C6	C15	C24				
	J. Gwinner	M. Lefebvre	F. Lara				
16:20 - 16:50	C7	C16	C25				
	C. Gutiérrez	M. López	G. Sotomayor				
16:50 - 17:10	COFEE BREAK						
17:10 - 17:40	C8	C17	<b>P6</b> (17:10 - 18:00)				
	L.M. Graña	J.E. Martínez-Legaz	R.I. Boț				
17:40 - 18:30	P2	P4	Ceremony Social Dinner				
	A. lusem	F. Flores-Bazán	18:00				

### **Plenaries**

- **P1** Aris Daniilidis, Centro de Modelamiento Matemático, Chile "Convex and semi-algebraic paradigms in structural optimization: dynamical considerations"
- **P2** Alfredo N. Iusem, Instituto de Matemática Pura e Aplicada, Brasil "On the quadratic eigenvalue complementarity problem"
- P3 Benar Fux Svaiter, Instituto de Matemática Pura e Aplicada, Brasil TBA
- **P4** Fabián Flores-Bazán, Universidad de Concepción, Chile "Joint-range convexity for a pair of inhomogeneous quadratic functions with applications"
- P5 Hélène Frankowska, CNRS-Institue Mathématique de Jussie, France "Second Order Necessary Optimality Conditions in Optimal Control"
- P6 Radu Ioan Boţ University of Vienna, Austria "A Douglas-Rachford type algorithm for nonsmooth convex optimization problems with complex structures"
- **P7** Sjur Didrik Flam, University of Bergen, Norway *"Reaching market equilibrium"*
- **P8** Stephen Simons, University od California Santa Barbara, USA *"SN spaces "densities" and maximal monotonicity"*

## **Conferences and Contributed Talks**

#### Monday

- **C1** Michel Théra, U. of Limoges "On extended versions of Dancs-Hegedüs-Medvegyev's Fixed Point Theorem"
- **C2** Jean-Bernard Baillon, Université Paris 1 Panthéon-Sorbonne, France "Asymptotic behavior of compositions of under-relaxed nonexpansive operators"
- **C3** Genaro López-Acedo, Universidad de Sevilla, España "Chebyshev sets in geodesic spaces"
- C4 Rafael Correa, Centro de Modelamiento Matemático, Chile "On Klee-Saint Raymond's Characterization of convexity"
- **C5** Ernö Robert Csetnek, University of Vienna, Austria "A forward-backward dynamical approach to the minimization of the sum of a nonsmooth convex with a smooth nonconvex function"
- **C6** Joachim Gwinner, IM University of the Federal Army, Germany "From equilibria and variational inequalities to nonmonotone contact in continuum mechanics"
- **C7** César Gutiérrez, Universidad de Valladolid, España "Scalarization in ordered spaces: from vector optimization to set optimization"
- **C8** L. M. Graña Drummond, Universidade Federal do Rio de Janeiro, Brasil "On balanced Pareto optima and descent methods for multicriteria"

### Tuesday

- **C9** Abderrahim Hantoute, Centro de Modelamiento Matemático, Chile *"Formulas for the normal cone to sublevel sets using ε-subdifferential"*
- **C10** Taron Zakaryan, Centro de Modelamiento Matemático, Chile "(Sub)differentiability of the infimal convolutions and the minimal time function"
- **C11** Julio López, Universidad Diego Portales, Chile "A feasible direction algorithm for nonlinear second-Order cone complementarity problems"
- **C12** Ulrich Kohlenbach, Technische Universität Darmstadt, Germany *"Effective bounds in convex optimization by logical methods"*
- C13 Alain Quilliot, ISIMA, France "Old Problems, New Paradigms in Operations Research"
- **C14** Marc Lassonde, Université des Antilles et de la Guyane, France *"On the subderivative-subdifferential duality"*
- **C15** Mario Lefebvre, École Polytechnique de Montréal, Canadá *"Using symmetry to solve LQG homing problems in one and two dimensions"*
- **C16** Marco A. López, University of Alicante, Spain "Applying outer limit of sudifferentials to estimate calmness moduli of inequality systems"
- **C17** Juan Enrique Martínez-Legaz, Universitat Autò noma de Barcelona "A general nonconvex multiduality principle"

#### Wednesday

- **C18** W. Sosa, UCB, Brasil "Remarks on convex sets satisfying QBBAM property"
- **C19** Juan Peypouquet, Univesidad Técnica Federico Santa Maria, Chile "Fast Convergence of an Inertial Gradient-like System with Vanishing Viscosity Presenting"
- **C20** Marc Quincampoix, CNRS-UMR6205, Université de Brest, France "Vanishing discount limit for nonexpansive optimal control"
- **C21** Roger Behling, UFSC-Blumenau, Brazil "A constrained-projected Levenberg-Marquardt method under the constrained error bound condition"
- **C22** Rubén López, Universidad Católica de la Santísima Concepción, Chile "On set optimization problems"
- **C23** Jan-J. Rückmann, University of Bergen, Norway "On proper efficiency in multiobjective semi-infinite optimization"
- **C24** Felipe Lara, Universidad de Concepción, Chile "Asymptotic analysis for quasiconvex functions"
- **C25** Guina Sotomayor Alzamora, IMPA, Brasil "Bilevel optimization for the hierarchical hub location and flow allocation problem"

### Thursday

- **C26** Russell Luke, Universität Götinge, Germany "A survey of results on linear convergence for iterative proximal algorithms in nonconvex settings"
- **C27** Michel De Lara, CERMICS, Ecole des Ponts ParisTech, Francia *"Kinked, flat or curved: how the shape of the action set makes information shine"*
- **C28** Valentina Sessa, Instituto de Matemática Pura e Aplicada, Brasil *"The second-order cone quadratic eigenvalue complementarity problem"*
- **C29** Welington de Oliveira, UERJ and IMPA, Brasil "Nonsmooth optimization algorithms for primal-dual problems via augmented Lagrangians"
- **C30** Christiane Tammer, Martin Luther University Halle-Wittenberg, Germany "On set-valued optimization problems with variable ordering structure"

### Abstracts

Formulas for the normal cone to sublevel sets using  $\varepsilon\text{-}$  subdifferential

Abderrahim Hantoute CMM, Chile ahantoute@dim.uchile.cl

The gradient of a continuously differentiable function  $\Phi \in C^1(\mathbb{R}^N)$  at a given point  $\overline{x} \in \mathbb{R}^N$  is orthogonal to its level surface  $[\Phi = \Phi(\overline{x})]$  whenever  $\overline{x}$  is not a critical point. Many generalization can be found in the literature, namely, the recent result due to Thibault and Cabot [1] states that for a lower semicontinious convex function  $\Phi : X \to \mathbb{R} \cup \infty$  with X a reflexive Banach space and  $\overline{x} \in dom\Phi$  we have, without any qualification condition,

$$N_{[\Phi \le \Phi(\overline{x})]}(\overline{x}) = \limsup_{x \to \overline{x}} R_+ \partial \Phi(x)$$

The aim of this paper is to give another formula written by means of the  $\varepsilon$ -subderivatives of  $\Phi$  at the point  $\overline{x}$ . The main feature of our formula is that it only depends on the reference point  $\overline{x}$  rather than nearby points, and, as the formula above, it does not require any qualification condition. Moreover, the use of the  $\varepsilon$ -subdifferential allows us to establish this characterization in the general setting of locally convex spaces. We shall also show how to derive some classical results on the subject.

#### References

- A. CABOT, L. THIBAULT, Sequential formulae for the normal cone to sublevel sets, *Trans. of the AMS* 366 : 6591–6628, 2014.
- [2] A. HANTOUTE, A. SVENSSON, Formulas for the normal cone to sublevel sets using  $\varepsilon$ -subdifferential, working paper, 2015.

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OLD PROBLEMS, NEW PARADIGMS IN OPERATIONS RESEARCH

### Alain Quilliot ISIMA, France

C13 - Tue 6 14:30h

C9 - Tue 6 09:00h

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Operations Research is an old scientific field, born, in the years 40's, and which was deeply impregnated of the centralized paradigm and the Mathematical programming framework.

While considering here a simple reference scheduling problem, we show how techniques and approaches have been evolving and how fundamental changes in both technologies and socio-economics lead to reformulation of standard decision problems and to a deep renewal of both concepts and methods.

# On the Quadratic Eigenvalue Complementarity Problem

Alfredo N. lusem IMPA, Brasil iusp@impa.br

P2 - Mon 5 17:40h

*Keywords:* Eigenvalue Problems, Complementarity Problems, Nonlinear Programming, Global Optimization.

Mathematics Subject Classification: 65F15, 90C33, 90C30, 90C26

We introduce several new results on the Quadratic Eigenvalue Complementarity Problem (QEiCP), focusing on the nonsymmetric case, i,e, without making symmetry assumptions on the matrices defining the problem. First we establish a new sufficient condition for existence of solutions of this problem, which is somewhat more manageable than previously existent ones. This condition works through the introduction of auxiliary variables which leads to the reduction of QEiCP to an Eigenvalue Complementarity Problem (EiCP) in higher dimension. Hence, this reduction suggests a new strategy for solving QEiCP, which is also analyzed in the paper. We also present an upper bound for the number of solutions of QEiCP and exhibit some examples of instances of QEiCP whose solution set has large cardinality, without attaining though the just mentioned upper bound. We also investigate the numerical solution of the QEiCP by exploiting a nonlinear programming and a variational inequality formulations of using these formulations for solving the QEiCP in practice.

Joint work with Carmo Brás and Joaquim J. Júdice.

CONVEX AND SEMI-ALGEBRAIC PARADIGMS IN STRUCTURAL OPTIMIZA-TION: DYNAMICAL CONSIDERATIONS

### Aris Daniilidis

P1 - Mon 5 10:50h

CMM, Chile arisd@mat.uab.cat

In this talk, we discuss asymptotic properties of orbits of dynamical systems that are naturally linked to variational analysis and optimization: subgradient systems, proximal algorithms, convex foliations, sweeping process. Curve selection lemma (in the semi-algebraic paradigm) and self-contractedness (in the convex paradigm) will be the cornerstones of this study.

# Scalarization in ordered spaces: From vector optimization to set optimization

César Gutiérrez

C7 - Mon 5 16:20h

C30 - Thu 8 14:30h

Universidad de Valladolid, España cesargv@mat.uva.es

Scalarization processes are recognized as one of the most powerful tools to study and solve vector optimization problems. Roughly speaking, they associate with the vector optimization problem a scalar optimization problem in such a way that the solutions of the scalar problem are solutions of the vector problem, and reciprocally. In this talk, a suitable and very simple approach is introduced for dealing with scalarization processes in arbitrary ordered spaces. It is illustrated by well-known scalarization schemes in vector optimization and finally, it is applied to characterize solutions of set optimization problems.

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TBA			
Benar Fux Sv IMPA, Brasil	vaiter		P3 - Tue 6 10:50h
benar@impa.b:	r		

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ON SET-VALUED OPTIMIZATION PROBLEMS WITH VARIABLE ORDERING STRUCTURE

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Christiane Tammer Martin Luther University Halle-Wittenberg, Germany. christiane.tammer@mathematik.uni-halle.de

We introduce and investigate an optimality concept for set-valued optimization problems with variable ordering structure. In our approach, the ordering structure is governed by a set-valued map acting between the same spaces as the objective multifunction. Necessary optimality conditions for the proposed problem are derived in terms of Bouligand and Mordukhovich generalized differentiation objects.

Joint work with Marius Durea and Radu Strugariu. **Keywords:** Nondomination property, Pareto optimization, variable ordering structure, openness for sum-multifunction, necessary optimality conditions

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# A FORWARD-BACKWARD DYNAMICAL APPROACH TO THE MINIMIZATION OF THE SUM OF A NONSMOOTH CONVEX WITH A SMOOTH NONCONVEX FUNCTION

#### Ernö Robert Csetnek

C5 - Mon 5 15:20h

University of Vienna, Austria ernoe.robert.csetnek@univie.ac.at

We address the minimization of the sum of a proper, convex and lower semicontinuous with a (possibly nonconvex) smooth function from the perspective of an implicit dynamical system of forward-backward type. The latter is formulated by means of the gradient of the smooth function and of the proximal point operator of the nonsmooth one. The trajectory generated by the dynamical system is proved to asymptotically converge to a critical point of the objective, provided a regularization of the latter satisfies the Kurdyka-Łojasiewicz property. Convergence rates for the trajectory in terms of the Łojasiewicz exponent of the regularized objective function are also provided.

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# JOINT-RANGE CONVEXITY FOR A PAIR OF INHOMOGENEOUS QUADRATIC FUNCTIONS WITH APPLICATIONS

P4 - Tue 6 17:40h

Fabián Flores-Bazán Universidad de Concepción, Chile fflores@ing-mat.udec.cl

The Dines theorem shows the convexity of the joint-range for pair of homogeneous quadratic functions. Since the same result fails when the quadratic functions are inhomogeneous, we characterize those directions we must add to the joint-range in order to get convexity. Its connection to quadratic minimization problems under a single equality quadratic constraint is discussed, among them the validity of Strong duality property.

The results were obtained jointly with Felipe Opazo.

#### References

 FLORES-BAZÁN F. & OPAZO, F., Joint-range convexity for a pair of inhomogeneous quadratic functions and applications to QP. arXiv:1508.01612, 2015.

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#### Asymptotic analysis for quasiconvex functions

#### Felipe Lara

Universidad de Concepción, Chile felipelara@udec.cl C24 - Wed 7 15:50h

We endeavour to answer the question. Is the usual asymptotic (reces- sion) function appropriate to describe the behaviour of a quasiconvex func- tion at the infinity?. To that end, we develop several properties of the incident asymptotic function proposed by J. P. Penot in [5], a new formula for com- puting the incident asymptotic function is given without any convexity as- sumption. We analyse and compared this asymptotic function with the usual and with various other attempts for quasiconvex asymptotic functions pre- sented in the literature. Finally, applications to the scalar and multiobjective optimization problem are also provide.

#### References

- AMARA CH. Directions De Majoration D'une Fonction Quasiconvexe et Applications, Serdica Math. J., 24 289–306, 1998.
- [2] AUSLENDER A. AND TEBOULLE M. Asymptotic cones and functions in optimization and variational inequalities, *Springer Verlag, New York, Berlin,* 2003.
- [3] FLORES-BAZÁN F., HADJISAVVAS N. AND LARA F. Second order asymptotic analysis: Basic Theory, J. Of Convex Anal. 22, 2015.
- [4] FLORES-BAZÁN F., AND VERA C. Maximazing and minimazing quasiconvex functions: Related Properties, existence and optimality conditions via radial epiderivates, *Published online in J. Global Optimization*, 2015.
- [5] PENOT J.P. What is quasiconvex analysis?, Optimization, 47 35–100, 2000.

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CHEBYSHEV SETS IN GEODESIC SPACES

Genaro López-Acedo Universidad de Sevilla, España glopez@us.es C3 - Mon 5 14:00h

The following result, see [1], about the structures of Chebyshev sets in Hilbert spaces summarizes the main properties of the metric projection in this setting.

**Theorem 1** Let *H* be a Hilbert space and  $C \subseteq X$  a Chebyshev set. Each of the following statements is equivalent to *C* being convex:

- (1) C is weakly closed (Klee);
- (2) C is approximatively compact (Efimov-Stechkin);

- (3)  $P_C$  is continuous (Vlasov);
- (4)  $P_C$  is radially continuous (Vlasov);
- (5) for every  $x \in H \setminus C$ ,

$$\lim_{\varepsilon \to 0^+} \frac{\mathsf{dist}(x_\varepsilon, C) - \mathsf{dist}(x, C)}{\|x_\varepsilon - x\|} = 1,$$

where  $x_{\varepsilon} = x + \varepsilon [x - P_C(x)]$  (Vlasov );

- (6) C is a sun (Efimov-Stechkin);
- (7)  $P_C$  is nonexpansive (Phelps).

The aim of this talk is to carry out a systematic study of properties of Chebyshev sets in the setting of geodesic spaces with emphasis on the influence of curvature on such properties. More precisely, we analyze the validity of Theorem 1 in spaces of bounded curvature in the sense of Alexandrov.

#### References

[1] F. Deutsch, Best Approximation in Inner Product Spaces, Springer-Verlag, New York, 2001.

BILEVEL OPTIMIZATION FOR THE HIERARCHICAL HUB LOCATION AND FLOW ALLOCATION PROBLEM

C25 - Wed 7 16:20h

Guina Sotomayor Alzamora IMPA, Brasil guinas@gmail.com

The hub location and the flow allocation problem aims to find points of concentration in a network flow so that the sum of the flow connections shared between the network points is minimized. In this work, a bilevel op- timization model for the fundamental hub location problem with simple flow allocation in a network is presented. The proposed model integrates the hi- erarchical decisions on locating regional and central hubs, at the first level, with the allocation of flows between points of the network, at the second level, aiming at minimizing the distances traveled by the flows. Computa- tional experiments with the proposed model are conducted for instances of the American and the Brazilian air transportation systems, using the CPLEX solver and a own software implemented with graphical features. Preliminary numerical results show the potential of the bilevel model proposed here.

Joint work with Fernanda M.P. Raupp.

#### References

- GUINA SOTOMAYOR ALZAMORA AND FERNANDA M.P. RAUPP. Modelo de otimização em dois níveis para localização hierárquica de hubs. In Simpósio Brasileiro de Pesquisa Operacional (SBPO), PE, 2015.
- [2] GUINA SOTOMAYOR ALZAMORA AND FERNANDA M.P. RAUPP. Modelo integrado para localização hierárquica de hubs em redes de fluxos. In Simpósio de Pesquisa Operacional e Logística da Marinha (SPOLM), RJ, 2015.
- [3] JAMES F CAMPBELL AND MORTON E O'KELLY. Twenty-five years of hub location research. *Transportation Science*, 46:153–169, 2012.
- [4] HANDE YAMAN. The hierarchical hub median problem with single assignment. *Transportation Research Part B*, 43:643–658, 2009.

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SECOND ORDER NECESSARY OPTIMALITY CONDITIONS IN OPTIMAL CONTROL

Hélène Frankowska CNRS-IMJ, Francia helene.frankowska@imj-prg.fr

P5 - Wed 7 10:50h

We discuss notions of second order tangents and normals to sets and apply them to investigate second order optimality conditions for the Mayer problem in optimal control. It is well known that with every optimal trajectory it is possible to associate a second order necessary optimality condition in the integral form whenever the control set is described by inequality and equality constraints. Instead we work with a general control set to get a similar inequality using second order tangents. We show how this integral condition leads to various pointwise conditions in quite general setting.

#### References

- FRANKOWSKA H. & TONON D. (2013) Pointwise second-order necessary optimality conditions for the Mayer problem with control constraints, SIAM Journal on Control and Optimization, 51, 3814–3843
- [2] H. FRANKOWSKA AND N. OSMOLOVSKII, Second-order necessary optimality conditions for the Mayer problem subject to a general control constraint, in Analysis and Geometry in Control Theory and its Applications, vol. 12 of Springer INDAM series, Springer Verlag, 2015.
- [3] H. FRANKOWSKA AND D. HOEHENER, Jacobson type necessary optimality conditions for general control systems, submitted.

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ON PROPER EFFICIENCY IN MULTIOBJECTIVE SEMI-INFINITE OPTIMIZA-TION

C23 - Wed 7 15:20h

C2 - Mon 5 10:20h

Jan-J. Rückmann University of Bergen, Norway Jan-Joachim.Ruckmann@ii.uib.no

We consider multiobjective semi-infinite optimization problems which are defined by finitely many objective functions and infinitely many inequality constraints in a finite-dimensional space. We discuss constraint qualifications as well as necessary and sufficient conditions for locally weakly efficient solutions. Furthermore, we generalize two concepts of properly efficient solutions to the semi-infinite setting and present corresponding optimality conditions.

This is a joint work together with Francisco Guerra Vázquez from the Universidad de las Américas, Puebla, México.

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#### Asymptotic behavior of compositions of under-relaxed nonexpansive operators

Jean-Bernard Baillon Université Paris 1 Panthéon-Sorbonne, France baillon@univ-paris1.fr

In general there exists no relationship between the fixed point sets of the composition and of the average of a family of nonexpansive operators in Hilbert spaces. In this paper, we establish an asymptotic principle connecting the cycles generated by under-relaxed compositions of nonexpansive operators to the fixed points of the average of these operators. In the special case when the operators are projectors onto closed convex sets, we prove a conjecture by De Pierro which has so far been established only for projections onto affine subspaces.

This work was maded with Patrick L. Combettes and Roberto Cominetti

#### References

 BAILLON JB., COMBETTES PL. & COMINETTI R., Asymptotic behavior of compositions of under-relaxed nonexpansive operators, *Journal of Dynamics and Games* volume 1 (3): 331 – 346, 2014.

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#### FROM EQUILIBRIA AND VARIATIONAL INEQUALITIES TO NONMONOTONE CONTACT IN CONTINUUM MECHANICS

#### Joachim Gwinner

C6 - Mon 5 15:50h

IM - University of the Federal Army, Germany
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In this talk we start from well-known existence results for abstract equilibria problems, for linear and for pseudo-monotone variational inequalities in reflexive Banach spaces. We discuss the necessity of the involved coerciveness conditions and their relationship. Then we combine Mosco convergence of convex closed sets with an approximation of pseudo-monotone bifunctions and provide a convergent approximation procedure for pseudo-monotone variational inequalities in reflexive Banach spaces. Since hemivariational inequalities in linear elasticity are pseudomonotone, our approximation method applies to nonmonotone contact problems. We sketch how methods of nonsmooth optimization together with finite element approximation lead to an efficient numerical solution method for these nonconvex nondifferentiable optimization problems. To illustrate our theory, we give a numerical example of a 2D linear elastic block under a given nonmonotone contact law.

This talk is based on joint work with Dr. N. Ovcharova [2]

*Keywords:* Pseudo-monotone bifunction, coerciveness condition, hemivariational inequality,nonmonotone contact, finite element discretization, regularization, nonsmooth optimization methods

References

- [1] J. GWINNER, A NOTE ON THE LIONS-STAMPACCHIA. Theorem for Variational Inequalities Applied to Singular. *Integral Operator Theory*, to appear.
- [2] J. GWINNER AND N. OVCHAROVA. From Solvability and Approximation of Variational Inequalities to Solution of Nondifferentiable Optimization Problems. *Contact Mechanics, Optimization* 64 (2015), 1683 -1702.

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#### A GENERAL NONCONVEX MULTIDUALITY PRINCIPLE

C17 - Tue 6 17:10h

Juan Enrique Martínez-Legaz Universitat Autònoma de Barcelona JuanEnrique.Martinez.Legaz@uab.cat

The main result in this joint work with F. Bonenti and R. Riccardi presents an arbitrary collection of mutually dual nonconvex optimization problems, as well as a characterization of their global optimal solutions. As immediate consequences of our general multiduality principle, we obtain Toland-Singer duality theorem and an analogous result involving generalized perspective functions.

FAST CONVERGENCE OF AN INERTIAL GRADIENT-LIKE SYSTEM WITH VANISHING VISCOSITY PRESENTING

#### Juan Peypouquet

C19 - Wed 7 9:30h

C11 - Tue 6 10:20h

Universidad Técnica Federico Santa Maria, Chile juan.peypouquet@usm.cl

We study the fast convergence of the trajectories of a second-order gradient-like system with a vanishing viscosity coefficient depending on a parameter. When the underlying potential has minimizers, each trajectory converges weakly to one of them. Strong convergence occurs in various practical situations. Surprisingly, in the strongly convex case, convergence is arbitrarily fast depending on the values of the parameter. When the solution set is empty, the minimizing property still holds, but the rapid convergence of the values may not be satisfied. Time discretization of this system provides new fast converging algorithms, expanding the field of rapid methods for structured convex minimization.

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# A FEASIBLE DIRECTION ALGORITHM FOR NONLINEAR SECOND-ORDER CONE COMPLEMENTARITY PROBLEMS

Julio López Universidad Diego Portales, Chile julio.lopez@udp.cl

In this work, we present a new feasible direction algorithm for solving nonlinear secondorder cone complementarity problems (SOCCPs). Given an interior point to the feasible set, the proposed algorithm computes a fea- sible and descent direction for an appropriate potential function. The search direction is computed by solving a Newton's system modified. Then, a line search along the search direction finds a new feasible point that has a lower value of the potential function. Repeating this process, the algorithm gener- ates a feasible sequence with a monotone decreasing of the potential func- tion. Under mild assumptions we prove global convergence of the present algorithm. Numerical testing with several test problems is carried out and reported.

Joint work with Miguel Carrasco and Héctor Ramírez.

#### References

- ALIZADEH F. AND GOLDFARB D. Second-order cone programming, *Math. Program.*, 95 (1), Ser. B:3–51, 2003.
- [2] FUKUSHIMA M., LUO Z.Q. AND TSENG P., Smoothing functions for second order cone complementarity problems, SIAM J. Optim., 12:436–460, 2001.

- [3] HAYASHI S., YAMASHITA N. AND FUKUSHIMA M., A combined smoothing and regularization method for monotone second-order cone complementarity problems, SIAM J. Optim., 15:593-615, 2005.
- [4] J. HERSKOVITS AND S.R. M AZORCHE. A feasible directions algorithm for nonlinear complementarity problems and applications in mechanics, *Struct. Multidiscip. Optim.*, **37** (5): 435–446, 2009.

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### On balanced Pareto optima and descent methods for multicriteria

Luis M. Graña Drummond UFRJ, Brasil bolsigeno@gmail.com

We present first order characterizations of the efficient frontier for smooth multiobjective unconstrained problems. We also introduce the notion of balanced Pareto optimal solutions for general multicriteria problems and study some of its properties. Finally, we present a framework for multiobjective descent algorithms, which includes as particular cases the vectorvalued versions of the steepest descent, the projected gradient and the Newton methods.

#### References

- [1] GRAÑA DRUMMOND L. M., On balanced Pareto optima, In progress. 2015.
- [2] FUKUDA ELLEN H., GRAÑA DRUMMOND L. M., A survey on multiobjective descent methods, Pesquisa Operacional 34 (3): 585 – 620, 2014.
- [3] GRAÑA DRUMMOND L. M., SVAITER B. F., Métodos de descida para otimização multiobjetivo, 30° Colóquio Brasileiro de Matemática, Publicações Matemáticas, Rio de Janeiro, 2015.

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#### On the subderivative-subdifferential duality

#### Marc Lassonde

C14 - Tue 6 15:20h

C8 - Mon 5 17:10h

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We provide a formula relating the radial (lower Dini) subderivative to appropriate subdifferentials for arbitrary extended real-valued lower semicontinuous functions. We then introduce the class of directionnaly upper semismooth functions for which the radial subderivative at a given point of their domain can be fully expressed in terms of any subdifferential of the function at neighbouring points. This class includes the proper lower semicontinuous (directionnally, approximately) convex functions, the lower- $C^1$  functions, the locally Lipschitzian Clarke regular functions, the Mifflin semismooth functions, etc. Finally, we define a large class of functions which can be recovered (up to a constant) from a subdifferential on a convex open set, namely the class of essentially directionnaly upper semismooth functions. We then encompass most of the previous numerous works dealing with subdifferential determination of functions, see some references below.

#### References

- BORWEIN, JONATHAN M. & MOORS, WARREN B., Null sets and essentially smooth Lipschitz functions, SIAM J. Optim. 8 (2): 309–323, 1998.
- [2] MOREAU J.-J., Proximité et dualité dans un espace hilbertien, *Bull. Soc. Math. France*, **93**: 273–299, 1965.
- [3] ROCKAFELLAR R. T., Characterization of the subdifferentials of convex functions, Pacific J. Math., 17: 497–510, 1966.
- [4] ROCKAFELLAR R. T., On the maximal monotonicity of subdifferential mappings, *Pacific J. Math.*, 33: 209–216, 1970.
- [5] THIBAULT L. & ZAGRODNY D., Subdifferential determination of essentially directionally smooth functions in Banach space, *SIAM J. Optim.*, **20** (5): 2300–2326, 2010.

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#### APPLYING OUTER LIMIT OF SUDIFFERENTIALS TO ESTIMATE CALMNESS MODULI OF INEQUALITY SYSTEMS

C16 - Tue 6 16:20h

Marco A. López University of Alicante, Spain marco.antonio@ua.es

We are mainly concerned with functions which are the maximum of a finite amount of continuously differentiable functions of n real variables, paying special attention to the case of polyhedral functions. For these max-functions, we present some results about outer limits of subdifferentials. When confined to the convex case, these results are applied to derive certain lower and upper estimates for the calmness modulus of convex inequality systems. Illustrative examples are given. The results presented in this talk can be found in [1].

#### References

 CANOVAS M.J., HENRION R., LOPEZ M.A., & PARRA J., Outer limit of subdifferentials and calmness moduli in linear and nonlinear programming, *J. Optim. Theory Appl.*, 2015, published online (DOI: 10.1007/s10957-015-07 93-x).

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#### VANISHING DISCOUNT LIMIT FOR NONEXPANSIVE OPTIMAL CONTROL

#### Marc Quincampoix

C20 - Wed 7 10:20h

CNRS-UMR6205, Université de Brest, France marc.quincampoix@univ-brest.fr

A classical problem in ergodic control consists of studying the limit behavior of the optimal value of a discounted cost functional with infinite horizon as the discount factor  $\lambda$  tends to zero. In the literature, this problem has been addressed under various conditions ensuring that the rescaled value function converges uniformly to a constant limit. The main goal of this paper is to study this problem without such conditions, so that the aforementioned limit needs not be constant. We show that, for nonexpansive dynamics and radially nondecreasing Hamiltonians, the above convergence turns out to be monotone and the uniform limit can be characterized as the maximal subsolution of a certain Hamilton-Jacobi equation. This allows not only to prove the convergence of the values but also this enables us to give new explicit representation formulas of the limits for control systems and for differential games.

#### References

- Cannarsa P. & Quincampoix M., Vanishing discount limit for nonexpansive optimal control, SIAM Journal of Control and Opti. 53, 789-814, (2015)
- [2] Quincampoix M. & Renault, J. On existence of a limit value in some non expansive optimal control problems, *SIAM Journal of Control and Opti.* **49**, 2118-2132, (2012)

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# USING SYMMETRY TO SOLVE LQG HOMING PROBLEMS IN ONE AND TWO DIMENSIONS

Mario Lefebvre

C15 - Tue 6 15:50h

École Polytechnique de Montréal, Canadá mario.lefebvre@polymtl.ca

We consider the stochastic optimal control problem known as LQG homing, in which one tries to minimize or maximize the time spent by a diffusion process inside a given region. In some cases, it is possible to reduce the control problem to a purely probabilistic one. Moreover, when symmetry can be used, exact and explicit expressions for the optimal control can be deduced from the solution to a first-order non- linear differential equation, rather than a second-order equation. In the general case, we show how to obtain good approximate solutions. Examples in one and two dimensions are presented.

Joint work with Foued Zitouni

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# KINKED, FLAT OR CURVED: HOW THE SHAPE OF THE ACTION SET MAKES INFORMATION SHINE

Michel De Lara CERMICS, Ecole des Ponts ParisTech, Francia delara@cermics.enpc.fr C27 - Thu 8 9:30h

We consider a decision maker who faces an uncertain state of Nature and chooses an action so as to maximize an expected payoff after observing a signal from an information structure. The value of information (VoI) is the difference of maximal expected payoffs with and without information.

How much information is worth depends jointly on the decisions at stake and the information provided. We characterize the Vol based on separate conditions on the information structure and the choices available (instrumental approach). Using tools from convex analysis, we are able to provide estimates for the Vol depending on the shape of the action set. We show that the Vol depends on how strong is the effect of information on choices:

- Highest in case of an indifference between actions (kinked); a "small piece" of information can have a large influence on the optimal decision.
- Mild when the decision problem is continuous (curved); in this case, the optimal decision for a given belief is "almost optimal" (envelope theorem) for close enough beliefs.
- Lowest when the optimal decision does not locally depend on the belief (flat); here, the decision maker is "hard to convince" to change decisions.

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# ON EXTENDED VERSIONS OF DANCS-HEGEDÜS-MEDVEGYEV'S FIXED POINT THEOREM

C1 - Mon 5 9:30h

Michel Théra U. of Limoges michel.thera@unilim.fr

This presentation is based on a recent joint work with Truong Q. Bao. It will provide an extension of Dancs-Hegedüs-Medvegyev's Fixed Point Theorem which not only unify several recent generalized versions of this theorem due to Khanh and Quy, the preorder principles established by Qiu, and the results obtained by Bao et al, but also further extend them to the setting of quasi-metric space. This feature allows us to obtain new applications to Ekeland's variational principle and Caristi's fixed point theorem.

#### References

 TRUONG Q. BAO. & THERA M., On Extended Versions of Dancs-Hegedüs-Medvegyev's Fixed Point Theorem, *Optimisation* volume dedicated to Franco Giannessi and Diethard Pallaschke arXiv:1508.01925v1. 123 – 321, 2014.

#### A DOUGLAS-RACHFORD TYPE ALGORITHM FOR NONSMOOTH CONVEX OPTIMIZATION PROBLEMS WITH COMPLEX STRUCTURES

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#### Radu Ioan Boț

P6 - Wed 7 17:10h

University of Vienna, Austria radu.bot@univie.ac.at

In this talk we address the solving of nonsmooth convex optimization problems with complex structures, by actually solving the corresponding system of optimality conditions, which involves sums of linear compositions of parallel sums of maximally monotone operators. We formulate an iterative scheme of Douglas-Rachford type that processes the set-valued maximally monotone operators via backward steps and the linear continuous operators via explicit forward steps, and analyze its convergence behaviour. The performances of the proposed algorithm are illustrated by numerical experiments in optimal location selection and convex risk minimization. The talk relies on [1,2].

#### References

- BOŢ R.I. & HENDRICH C., A Douglas-Rachford type primal-dual method for solving inclusions with mixtures of composite and parallel-sum type monotone operators, SIAM Journal on Optimization 23 (4): 2541 2565, 2013.
- [2] BOŢ R.I. & HENDRICH C., Convex risk minimization via proximal splitting methods, *Optimization Letters* **9** (5): 867 885, 2015.

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ON KLEE-SAINT RAYMOND'S CHARACTERIZATION OF CONVEXITY

#### Rafael Correa CMM, Chile rcorrea@dim.uchile.cl

C4 - Mon 5 14:30h

In [7] Saint Raymond observes that given a continuous function  $f: R \to R$  which satisfies  $\lim_{|x|\to+\infty} \frac{f(x)}{|x|} = +\infty$ , if it is non-convex there exists an affine function  $h \leq f$  such that the function f-h vanishes on a non-convex set. The work of Saint Raymond consists of showing that this fact characterizes convex functions:

**Theorem** [Saint Raymond's characterization theorem] Let X be a Banach space and  $f: X \to R \cup \{+\infty\}$  be a proper and weakly lower semiconinuous (lsc) function such that  $f - x^*$  is weakly infcompact for every  $x^* \in X^*$ . If the set  $argmin\{f - x^*\}$  is convex for all  $x^*$  in a convex dense subset D of  $X^*$ , then f is convex.

To prove this result [7, theorem 10], Saint Raymond uses classical tools of Banach space theory, more precisely, James' theorem [4, theorem 3.130 p.137] and Brouwer's fixed-point theorem for multifuncions [4, theorem 1 p. 523] among others. Another proof was more recently given by B. Ricceri [6, corollary 1], under the assumption that X is a reflexive Banach space.

In this work we use techniques of convex analysis to give an easy proof for a generalization of Saint Raymond's characterization of convexity, for functions defined in a (separated) locally convex space X in duality with another locally convex space Y. The epi-pointed assumption in our result, is verified under the hypothesis of Saint Raymond's theorem. This property has been successfully utilized recently ([2], [3], [5]) with the purpose of extending results, which for many years were known exclusively for Banach spaces or convex functions, to locally convex spaces and non-convex functions.

We have called this characterization *Klee-Saint Raymond* because in the framework of Hilbert spaces, Saint Raymond's characterization of convexity for w-lsc functions is equivalent to the famous characterization by Klee for the convexity of w-closed sets.

#### References

- [1] JEAN-PIERRE AUBIN, Mathematical methods of game and economic theory, volume 7 of Studies in Mathematics and its Applications. North-Holland Publishing Co., Amsterdam-New York, 1979.
- [2] J. BENOIST AND A. DANIILIDIS, Integration of Fenchel subdifferentials of epi-pointed functions, SIAM J. Optim., 12(3): 575–582, 2002.
- [3] R. CORREA, Y. GARCIA, & A. HANTOUTE, Integration formulas via the Fenchel subdifferential of nonconvex functions, *Nonlinear Anal.* **75** (3): 1188–1201, 2012.
- [4] M. FABIAN, P. HABALA, P. HAJEK, V. MONTESINOS, & VACLAV ZIZLER, Banach space theory. CMS Books in Mathematics/Ouvrages de Mathèmatiques de la SMC. Springer, New York, 2011.
- [5] P. PEREZ-AROS, Propiedades variacionales de funciones convexas desde el análisis epi-puntado. Engineering thesis, Universidad de Chile, 2014.
- [6] B. RICCERI, Singular points of non-monotone potential operators, Journal of Nonlinear and Convex Analysis, 16(6): 112–1129, 2015.
- [7] JJ. SAINT RAYMOND, Characterizing convex functions by variational properties, J. Nonlinear Convex Anal., 14(2): 253–262, 2013.

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# A CONSTRAINED-PROJECTED LEVENBERG-MARQUARDT METHOD UNDER THE CONSTRAINED ERROR BOUND CONDITION

C21 - Wed 7 14:00h

Roger Behling UFSC-Blumenau, Brazil roger.behling@ufsc.br

We consider the problem of solving a convex constrained system of nonlinear equations. In the last decade, projected and constrained Levenberg-Marquardt methods were studied in order to solve such problems under error bound type conditions, which do not imply solutions to be isolated. In our work, we assume a very general error bound condition and propose a constrained-projected Levenberg-Marquardt method whose hybridity combine the simplicity of the projected Levenberg-Marquardt method and the fast local convergence of the constrained one.

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#### ON SET OPTIMIZATION PROBLEMS

Rubén López Universidad Católica de la Santísima Concepción, Chile rlopez@ucsc.cl C22 - Wed 7 14:30h

In this talk we study optimization problems where the objective map is a set-valued map. These are the so-called set optimization problems. We study these problems by considering the set criterion proposed by Kuroiwa in 1998. To study these problems we employ an asymptotic method. To do this, we introduce a suitable notion of asymptotic map for set-valued maps. By using this notion of asymptotic map we obtain coercive existence results for set optimization problems.

Coauthor: Elvira Hernández (Universidad Nacional de Educación a Distancia, Madrid, Spain. E-mail: ehernandez@ind.uned.es)

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#### A SURVEY OF RESULTS ON LINEAR CONVERGENCE FOR ITERATIVE PROX-IMAL ALGORITHMS IN NONCONVEX SETTINGS

#### Russell Luke

C26 - Thu 8 9:00h

Universität Göttingen, Germany christiane.tammer@mathematik.uni-halle.de

For iterative methods in nonconvex optimization, a central question is when to stop. And when the decision has been made to stop, what is the relation, if any, between the point that the algorithm delivers and the desired solutions to the optimization problem? At the heart of answers to these questions is the theory of regularity, not only of the underlying functions and operators, but of the set of solutions, and, more generally, critical points. We survey progress over the last several years on sufficient conditions for local linear convergence of fundamental algorithms applied to nonconvex problems, and discuss challenges and prospects for further progress. The theory is local by nature and contains the convex case as an example where the local neighborhood extends to the whole space. The popular affine feasibility problem illustrates that the convex case is not the only instance where global guarantees of convergence of first-order algorithms to globally optimal solutions are possible, and that regularity of the objective function, in conjunction with the constraint structure, is key to global results.

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#### REACHING MARKET EQUILIBRIUM

P7 - Thu 8 10:50

P8 - Thu 10 15:20

Sjur Didrik Flåm University of Bergen, Norway sjur.Flaam@econ.uib.no

Direct exchange may equilibrate markets. Clearing prices can emerge by way of blockcoordinate, distributed programming - of adaptive, gradient type - fully driven by the agents themselves.

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 $\mathrm{SN}$  spaces "densities" and maximal monotonicity

Stephen Simons UCSB, USA stesim380gmail.com

Quasidensity is a concept that can be applied to subsets of  $E \times E^*$ , where E is a nonzero real Banach space. Every closed quasidense monotone set is maximally monotone, but there exist maximally monotone sets that are not quasidense. The graph of the subdifferential of a proper, convex lower semicontinuous function on E is quasidense. The graphs of certain subdifferentials of certain nonconvex functions are also quasidense. (This follows from joint work with Xianfu Wang.) The closed monotone and quasidense sets have a number of very desirable properties, including a sum theorem and a parallel sum theorem, and so quasidensity satisfies the ideal calculus rules. There are many conditions equivalent to the statement that a closed monotone set is quasidense. We start our discussion in the more general situation of  $r_L$ -dense subsets of SN spaces, where the notation is more concise.  $r_L$ -density also leads to a generalization of the Brezis–Browder theorem on linear relations.

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# (SUB)DIFFERENTIABILITY OF THE INFIMAL CONVOLUTIONS AND THE MINIMAL TIME FUNCTION

C10 - Tue 6 9:30h

Taron Zakaryan CMM, Chile tzakaryan@dim.uchile.cl

Let X be a Banach space whose norm is Gâteaux (Fréchet) differentiable. Cibulka and Fabian [1] proved that if the infimal convolution of a fairly general function on X and  $|| ||^2$  is strongly attained then it is Gâteaux (Fréchet) differentiable. We extend this result by showing that  $|| ||^2$  can be replaced by convex and coercive function which is Lipschitz on any bounded set. As a consequence, we obtain new result on differentiability of the minimal time function, an extension of the corresponding result of Colombo, Goncharov and Mordukhovich [2] to Banach space.

Joint work with A. Hantoute.

References

- CIBULKA R., FABIAN M., Attainment and (sub)differentiability of the infimal convolution of a function and the square of the norm, *J. Math. Anal. Appl.* 368 (2): 538 – 550, 2010.
- [2] COLOMBO G., GONCHAROV V.V., MORDUKHOVICH B.S., Well-Posedness of Minimal time Problems with constant Dynamics in Banach Spaces, *Set-Valued Anal.* 18: 349 – 372, 2010.

EFFECTIVE BOUNDS IN CONVEX OPTIMIZATION BY LOGICAL METHODSUlrich KohlenbachC12 - Tue 6 14:00hTechnische Universität Darmstadt, Germany

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In recent years methods from mathematical logic have been used to extract explicit effective bounds from prima facie noneffective proofs in analysis ('proof mining'). We will give an introduction to this methodology and apply it to obtain e.g. explicit approximate fixed point bounds as well as (in the boundedly compact case) rates of so-called metastability (in the sense of T. Tao) for iterations of compositions of metric projections in CAT( $\kappa$ )-spaces ( $\kappa > 0$ ) as well as (in the setting of Hilbert spaces) the proximal point algorithm. The metastability results make use of a general effective version of the strong convergence of (quasi-)Fejér monotone sequences in general boundedly compact metric spaces (some results are joined work with L. Leuştean and A. Nicolae).

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# THE SECOND-ORDER CONE QUADRATIC EIGENVALUE COMPLEMENTAR-ITY PROBLEM

#### Valentina Sessa IMPA, Brasil valsessa@impa.br

We investigate the solution of the Second-Order Cone Quadratic Eigenvalue Complementarity Problem (SOCQEiCP), which has a solution under reasonable assumptions on the matrices included in its definition. A Nonlinear Programming Problem (NLP) formulation of the SOCQEiCP is introduced. A necessary and sufficient condition for a stationary point (SP) of NLP to be a solution of SOCQEiCP is established. This condition indicates that, in many cases, the computation of a single SP of NLP is sufficient for solving SOCQEiCP. In order to compute a global minimum of NLP for the general case, we develop an enumerative method based on the Reformulation-Linearization Technique and prove its convergence. For computational effectiveness, we also introduce a hybrid method that combines the enumerative algorithm and a semi-smooth Newton method. Computational experience on the solution of a set of test problems demonstrates the efficacy of the proposed hybrid method for solving SOCQEiCP.

Joint work with A.N. lusem, J.J. Júdice and H.D. Sherali.

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# NONSMOOTH OPTIMIZATION ALGORITHMS FOR PRIMAL-DUAL PROBLEMS VIA AUGMENTED LAGRANGIANS

C29 - Thu 8 14:00h

C28 - Thu 8 10:20h

Welington de Oliveira UERJ and IMPA, Brasil wlo@impa.br

We consider nonlinear optimization problems with equality constraints and a dual schema via generalized augmented Lagrangians. Primal and dual convergence results are obtained by employing proximal bundle algorithms to the dual problem. Primal convergence is ensured even when the dual optimal set is empty. Moreover, depending on the proximal bundle variant, we show that primal convergence can be achieved even when subproblems defining the dual function are not (globally) solved up to optimality for every dual iterate.

Joint work with Claudia Sagastizábal.

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### REMARKS ON CONVEX SETS SATISFYING QBBAM PROPERTY

W. Sosa UCB, Brasil sosa@ucb.br

C18 - Wed 7 9:00h

In this manuscript we study convex sets for which every quadratic bounded below functions attains a minimum (QBBAM property). Some results about convex sets satisfying QBBAM are given.

Joint work with J.E. Martínez Legaz.

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